Review Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Finding or Evaluating an Integral

In Exercises 1–8, use the basic integration rules to find or evaluate the integral.

1. $\int x\sqrt{x^2 - 36}\,dx$

2. $\int xe^{x^2-1}\,dx$

3. $\int \frac{x}{x^2 - 49}\,dx$

4. $\int \frac{x}{\sqrt[3]{4 - x^2}}\,dx$

5. $\int_1^e \frac{\ln(2x)}{x}\,dx$

6. $\int_{3/2}^2 2x\sqrt{2x - 3}\,dx$

7. $\int \frac{100}{\sqrt{100 - x^2}}\,dx$

8. $\int \frac{2x}{x - 3}\,dx$

Using Integration by Parts

In Exercises 9–16, use integration by parts to find the indefinite integral.

9. $\int xe^{3x}\,dx$

10. $\int x^3e^x\,dx$

11. $\int e^{2x}\sin 3x\,dx$

12. $\int x\sqrt{x - 1}\,dx$

13. $\int x^2\sin 2x\,dx$

14. $\int \ln\sqrt{x^2 - 4}\,dx$

15. $\int x\arcsin 2x\,dx$

16. $\int \arctan 2x\,dx$

Finding a Trigonometric Integral

In Exercises 17–22, find the trigonometric integral.

17. $\int \cos^3(\pi x - 1)\,dx$

18. $\int \sin^2\frac{\pi x}{2}\,dx$

19. $\int \sec^4\frac{x}{2}\,dx$

20. $\int \tan\theta\sec^4\theta\,d\theta$

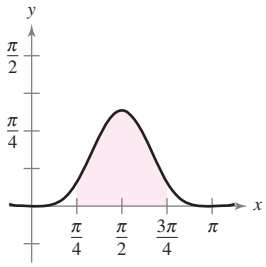
21. $\int \frac{1}{1 - \sin\theta}\,d\theta$

22. $\int \cos 2\theta(\sin\theta + \cos\theta)^2\,d\theta$

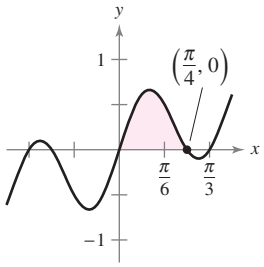
Area

In Exercises 23 and 24, find the area of the region.

23. $y = \sin^4 x$



24. $y = \sin 3x\cos 2x$



Using Trigonometric Substitution

In Exercises 25–30, use trigonometric substitution to find or evaluate the integral.

25. $\int \frac{-12}{x^2\sqrt{4 - x^2}}\,dx$

26. $\int \frac{\sqrt{x^2 - 9}}{x}\,dx, \quad x > 3$

27. $\int \frac{x^3}{\sqrt{4 + x^2}}\,dx$

28. $\int \sqrt{25 - 9x^2}\,dx$

29. $\int_0^1 \frac{6x^3}{\sqrt{16 + x^2}}\,dx$

30. $\int_3^4 x^3\sqrt{x^2 - 9}\,dx$

Using Different Methods

In Exercises 31 and 32, find the indefinite integral using each method.

31. $\int \frac{x^3}{\sqrt{4 + x^2}}\,dx$

(a) Trigonometric substitution

(b) Substitution: $u^2 = 4 + x^2$

(c) Integration by parts: $dv = \frac{x}{\sqrt{4 + x^2}}\,dx$

32. $\int x\sqrt{4 + x}\,dx$

(a) Trigonometric substitution

(b) Substitution: $u^2 = 4 + x$

(c) Substitution: $u = 4 + x$

(d) Integration by parts: $dv = \sqrt{4 + x}\,dx$

Using Partial Fractions

In Exercises 33–38, use partial fractions to find the indefinite integral.

33. $\int \frac{x - 39}{x^2 - x - 12}\,dx$

34. $\int \frac{5x - 2}{x^2 - x}\,dx$

35. $\int \frac{x^2 + 2x}{x^3 - x^2 + x - 1}\,dx$

36. $\int \frac{4x - 2}{3(x - 1)^2}\,dx$

37. $\int \frac{x^2}{x^2 + 5x - 24}\,dx$

38. $\int \frac{\sec^2\theta}{\tan\theta(\tan\theta - 1)}\,d\theta$

Integration by Tables

In Exercises 39–46, use integration tables to find or evaluate the integral.

39. $\int \frac{x}{(4 + 5x)^2}\,dx$

40. $\int \frac{x}{\sqrt{4 + 5x}}\,dx$

41. $\int_0^{\sqrt{\pi}/2} \frac{x}{1 + \sin x^2}\,dx$

42. $\int_0^1 \frac{x}{1 + e^{x^2}}\,dx$

43. $\int \frac{x}{x^2 + 4x + 8}\,dx$

44. $\int \frac{3}{2x\sqrt{9x^2 - 1}}\,dx, \quad x > \frac{1}{3}$

45. $\int \frac{1}{\sin\pi x\cos\pi x}\,dx$

46. $\int \frac{1}{1 + \tan\pi x}\,dx$

47. Verifying a Formula Verify the reduction formula

$$\int (\ln x)^n\,dx = x(\ln x)^n - n\int (\ln x)^{n-1}\,dx.$$

48. Verifying a Formula Verify the reduction formula

$$\int \tan^n x\,dx = \frac{1}{n - 1}\tan^{n-1}x - \int \tan^{n-2}x\,dx.$$

Finding an Indefinite Integral **In Exercises 49–56, find the indefinite integral using any method.**

49. $\int \theta \sin\theta \cos\theta \, d\theta$

50. $\int \frac{\csc\sqrt{2x}}{\sqrt{x}} \, dx$

51. $\int \frac{x^{1/4}}{1 + x^{1/2}} \, dx$

52. $\int \sqrt{1 + \sqrt{x}} \, dx$

53. $\int \sqrt{1 + \cos x} \, dx$

54. $\int \frac{3x^3 + 4x}{(x^2 + 1)^2} \, dx$

55. $\int \cos x \ln(\sin x) \, dx$

56. $\int (\sin\theta + \cos\theta)^2 \, d\theta$

Differential Equation **In Exercises 57–60, solve the differential equation using any method.**

57. $\frac{dy}{dx} = \frac{25}{x^2 - 25}$

58. $\frac{dy}{dx} = \frac{\sqrt{4 - x^2}}{2x}$

59. $y' = \ln(x^2 + x)$

60. $y' = \sqrt{1 - \cos\theta}$

Evaluating a Definite Integral **In Exercises 61–66, evaluate the definite integral using any method. Use a graphing utility to verify your result.**

61. $\int_2^{\sqrt{5}} x(x^2 - 4)^{3/2} \, dx$

62. $\int_0^1 \frac{x}{(x - 2)(x - 4)} \, dx$

63. $\int_1^4 \frac{\ln x}{x} \, dx$

64. $\int_0^2 xe^{3x} \, dx$

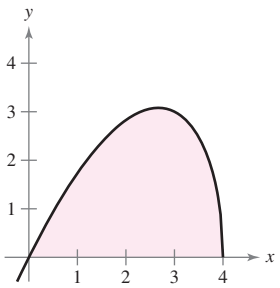
65. $\int_0^{\pi} x \sin x \, dx$

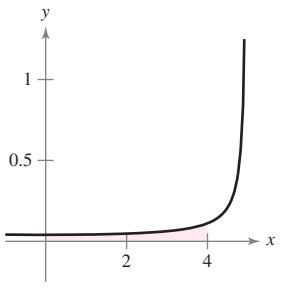
66. $\int_0^5 \frac{x}{\sqrt{4 + x}} \, dx$

Area **In Exercises 67 and 68, find the area of the region.**

67. $y = x\sqrt{4 - x}$

68. $y = \frac{1}{25 - x^2}$





Centroid **In Exercises 69 and 70, find the centroid of the region bounded by the graphs of the equations.**

69. $y = \sqrt{1 - x^2}, \quad y = 0$

70. $(x - 1)^2 + y^2 = 1, \quad (x - 4)^2 + y^2 = 4$

Arc Length **In Exercises 71 and 72, approximate to two decimal places the arc length of the curve over the given interval.**

Function	Interval
71. $y = \sin x$	$[0, \pi]$
72. $y = \sin^2 x$	$[0, \pi]$

Evaluating a Limit **In Exercises 73–80, use L'Hôpital's Rule to evaluate the limit.**

73. $\lim_{x \to 1} \frac{(\ln x)^2}{x - 1}$

74. $\lim_{x \to 0} \frac{\sin \pi x}{\sin 5\pi x}$

75. $\lim_{x \to \infty} \frac{e^{2x}}{x^2}$

76. $\lim_{x \to \infty} xe^{-x^2}$

77. $\lim_{x \to \infty} (\ln x)^{2/x}$

78. $\lim_{x \to 1^+} (x - 1)^{\ln x}$

79. $\lim_{n \to \infty} 1000\left(1 + \frac{0.09}{n}\right)^n$

80. $\lim_{x \to 1^+} \left(\frac{2}{\ln x} - \frac{2}{x - 1}\right)$

Evaluating an Improper Integral **In Exercises 81–88, determine whether the improper integral diverges or converges. Evaluate the integral if it converges.**

81. $\int_0^{16} \frac{1}{\sqrt[4]{x}} \, dx$

82. $\int_0^2 \frac{7}{x - 2} \, dx$

83. $\int_1^{\infty} x^2 \ln x \, dx$

84. $\int_0^{\infty} \frac{e^{-1/x}}{x^2} \, dx$

85. $\int_1^{\infty} \frac{\ln x}{x^2} \, dx$

86. $\int_1^{\infty} \frac{1}{\sqrt[4]{x}} \, dx$

87. $\int_2^{\infty} \frac{1}{x\sqrt{x^2 - 4}} \, dx$

88. $\int_0^{\infty} \frac{2}{\sqrt{x}(x + 4)} \, dx$

89. Present Value The board of directors of a corporation is calculating the price to pay for a business that is forecast to yield a continuous flow of profit of \$500,000 per year. The money will earn a nominal rate of 5% per year compounded continuously. What is the present value of the business

(a) for 20 years?

(b) forever (in perpetuity)?

(*Note:* The present value for t_0 years is $\int_0^{t_0} 500{,}000e^{-0.05t} \, dt$.)

90. Volume Find the volume of the solid generated by revolving the region bounded by the graphs of $y = xe^{-x}$, $y = 0$, and $x = 0$ about the x-axis.

91. Probability The average lengths (from beak to tail) of different species of warblers in the eastern United States are approximately normally distributed with a mean of 12.9 centimeters and a standard deviation of 0.95 centimeter (see figure). The probability that a randomly selected warbler has a length between a and b centimeters is

$$P(a \le x \le b) = \frac{1}{0.95\sqrt{2\pi}} \int_a^b e^{-(x - 12.9)^2/1.805} \, dx.$$

Use a graphing utility to approximate the probability that a randomly selected warbler has a length of (a) 13 centimeters or greater and (b) 15 centimeters or greater. (*Source: Peterson's Field Guide: Eastern Birds*)